Journal of Engineering Mathematics, Vol. 10, No. 4, October 1976
Noordhoff International Publishing–Leyden
Printed in The Netherlands

The mechano-caloric effect in thermo-elastic problems

E. L. ROETMAN

Department of Mathematics, University of Missouri, Columbia, Missouri, U.S.A.

(Received August 18, 1975)

SUMMARY
The inclusion of mechano-caloric effects has introduced important changes in the form of the equations for uncoupled heat transmission, whence to essentially distinct response predictions [7]. The current work is devoted to understanding the changes in the field equations for elastic systems and their harmonic solutions with the inclusion of mechano-caloric effects. Low frequency waves are uneffected, but high frequency waves are predicted to have increased propagation velocities. It is observed that the mechanical response at high frequencies can be determined from the dynamical equations alone if the adiabatic stress relations are used. Preliminary analysis for the propagation and dispersion at intermediate frequencies is given.

1. Introduction

Modification of the heat flux relation to account for finite signal speed has been the subject of a number of recent articles, [1]–[7]. In most of this work a heat flux relation generalizing Fourier's law is assumed in a form appropriate to produced a damped wave equation for the temperature

$$\partial_t^2\theta + a\partial_t\theta = b\nabla^2\theta + \ldots \tag{1}$$

whenever the system can be uncoupled. The extension of Fourier's law given by

$$\partial_t\mu = -\tau^{-1}(\mu + H\nabla\theta), \tag{2}$$

proposed by Vernotte [2], has been extensively investigated for a number of important applications by Tokuoka, see [6]. This generalization does not appear to have other modivation than that for the uncoupled case it yields a temperature distribution equation with the form (1). The generalization of Fourier's law of heat conduction for the flux $\boldsymbol{\mu}$ given by

$$\boldsymbol{\mu} = -H\nabla\theta - K\nabla p, \tag{3}$$

where θ is the temperature and p is the hydrostatic pressure, is discussed by Roetman in [7], [8] and is modivated in gas dynamics by mechano-caloric effects, de Groot and Mazur [9], Chapman and Cowling [10], Loyalka [11]. Here H is the heat conductivity and K is the mechano-caloric coefficient. This law is also consistent with the required coordinate invariance, Truesdell and Toupin [12]. It is of interest to investigate the significance of the mechano-caloric effect, heat flux law (3), in the description of thermo-mechanical behavior. We will show that the mechano-caloric coupling implies a damped wave equation for the temperature when the temperature equation can be uncoupled from the mechanic terms. We also show that this coupling influences the wave number and propagation velocity, in particular, the predicted limiting high frequency velocity for longitudinal waves is increased compared to the uncoupled case. We also observe that when uncoupling the heat equation the specific heat at constant stress should be used rather than the specific heat at constant strain.

The field equations for the continuity, momentum and energy for material response, without convective terms, are

$$\partial_t \rho + \rho \nabla \cdot \boldsymbol{v} = 0 \tag{4}$$

$$\rho \partial_t \boldsymbol{v} = \nabla \cdot \mathbf{T} + \rho \boldsymbol{f} \tag{5}$$

$$\rho \partial_t \varepsilon = \operatorname{tr}(\mathbf{TD}) + \omega - \nabla \cdot \boldsymbol{\mu} \tag{6}$$

where ρ, $\boldsymbol{v}$, $\boldsymbol{u}$, $\mathbf{T}$, $\boldsymbol{f}$, ε, $\mathbf{D}$, ω are the density, velocity field, displacement field, Piola–Kirchhoff stress tensor, body force density, internal energy density, rate of deformation tensor and the internal heat source density respectively and

$$\boldsymbol{v} = d_t \boldsymbol{u}, \ \ \mathbf{D} = \operatorname{Sym}(\nabla \boldsymbol{v}), \tag{7}$$

and $\operatorname{tr}(AB)$ is the convected inner product of tensors which in cartesian coordinates is given in terms of the matrices of the tensors by the trace of the matrix product of A and B. The special case of this system determined by the constitutive assumptions

$$\mathbf{T} = -p\mathbf{I}, \ \ \varepsilon = \varepsilon(\theta, v) \tag{8}$$

where $v = \rho^{-1}$ and $\mathbf{I}$ is the identity tensor, has been studied in [7], [8]. The assumptions in (8) do not imply a restriction to fluids, but for elastic materials, where $p = -\frac{1}{3} \operatorname{tr} \mathbf{T}$ is the mean stress, these assumptions are very restrictive.

In what follows we consider only a linearized theory for a restricted class of materials. For our purposes this is not a serious restriction since the essential features of the analysis and the conclusions do not depend significantly on this assumption. Thus, H, K and the Lamé constants, λ and μ, to be introduced below are all taken as constant.

2. Thermodynamics

Consistent with the assumptions of a homogeneous, isotropic, simple, elastic material, [12] and Carlson [13], the stress and internal energy are assumed to be functions of the strain $\mathbf{E}$,

$$\mathbf{E} = \operatorname{Sym}(\nabla \boldsymbol{u}), \tag{9}$$

and an equation of state is hypothesized in the form

$$\varepsilon = \varepsilon(\mathbf{E}, \eta) \tag{10}$$

where η is the entropy density, Coleman and Noll [14] and Coleman and Mizel [15]. The Clausius–Duhem inequality then gives

$$\theta = \partial_\eta \varepsilon(\mathbf{E}, \eta), \tag{11}$$

$$\mathbf{T}(\mathbf{E}, \eta) = \rho \partial_{\mathbf{E}} \varepsilon, \tag{12}$$

where θ is the temperature and η is the entropy density. Inverting (11), we write

$$\varepsilon = \varepsilon(\mathbf{E}, \theta) = \varepsilon(\mathbf{E}, \eta(\mathbf{E}, \theta)), \tag{13}$$

$$T(\mathbf{E}, \theta) = T(\mathbf{E}, \eta(\mathbf{E}, \theta)) \tag{14}$$

(we shall abuse the function notation using the subscript notation prevalent in thermodynamics to keep track of variable dependence.)

Define the heat capacities for constant strain and constant stress by

$$c_d = \theta \partial_\theta \eta)_\mathbf{E} = \partial_\theta \varepsilon)_\mathbf{E}, \ c_s = \theta \partial_\theta \eta)_\mathbf{T}. \tag{16}$$

Then

$$c_s = \mathrm{tr}\{[\rho \partial_\mathbf{E} \varepsilon)_\theta - \mathbf{T}] \partial_\theta \mathbf{E})_\mathbf{T}\} + c_d. \tag{17}$$

But

$$\rho \partial_\mathbf{E} \varepsilon)_\theta = \mathbf{T} + \rho \theta \partial_\mathbf{E} \eta)_\theta, \tag{18}$$

so that

$$c_s - c_d = \theta \, \mathrm{tr}\, [\partial_\mathbf{E} \eta)_\theta \partial_\theta \mathbf{E})_\mathbf{T}]. \tag{19}$$

From (12)

$$\partial_\mathbf{E} \eta)_\theta = -\partial_\theta (\rho^{-1} \mathbf{T})_\mathbf{E} = -\rho^{-1} \partial_\theta \mathbf{T})_\mathbf{E}; \tag{20}$$

the last equality follows from the constancy of $\mathbf{E}$. Returning to equation (6), one obtains for the energy, see also Carlson [13],

$$\rho c_d \partial_t \theta = -\nabla \cdot \boldsymbol{\mu} + \omega + \theta \, \mathrm{tr}\, [\partial_\theta \mathbf{T})_\mathbf{E} \partial_t \mathbf{E}]. \tag{21}$$

3. Mechanical equations

Reduction of the mechanical equations requires a constitutive relation and linearizations. Let

$$e = \mathrm{tr}\, \mathbf{E}. \tag{22}$$

Then for small strain the continuity equation reduces to

$$\rho = \rho_0 (1 - e), \tag{23}$$

Chadwick [16], while the density in the momentum and energy relations is taken to be the constant ρ_0.

Assume now a linear stress relation in the form

$$\mathbf{T} = \lambda e \mathbf{I} + 2\mu \mathbf{E} - m(\theta - \theta_0)\mathbf{I} \tag{24}$$

where λ, μ are the Lamé coefficients, $m = \alpha(3\lambda + 2\mu)$ and α is the coefficient of linear thermal expansion. The hydrostatic pressure, mean stress, $p = -\frac{1}{3} \mathrm{tr}\, \mathbf{T}$ is then

$$p = -\tfrac{1}{3}(3\lambda + 2\mu)e + m(\theta - \theta_0). \tag{25}$$

The diviatoric stress $\hat{\mathbf{T}}$ defined by

$$\mathbf{T} = -p\mathbf{I} + \hat{\mathbf{T}} \tag{26}$$

satisfies

$$\hat{\mathbf{T}} = 2\mu\hat{\mathbf{E}} \tag{27}$$

where $\hat{\mathbf{E}}$ is diviatoric strain.

On the other hand, the divergence of the momentum equation (5) gives

$$\rho_0\partial_t^2 e = -\nabla^2 p + \nabla\cdot(\nabla\cdot\hat{\mathbf{T}}) + \rho_0\nabla\cdot\boldsymbol{f}, \tag{28}$$

or

$$\nabla^2 p = -\rho_0\partial_t^2 e + 2\mu\nabla\cdot(\nabla\cdot\hat{\mathbf{E}}) + \rho_0\nabla\cdot\boldsymbol{f} \tag{29}$$

Also

$$\partial_\theta\mathbf{T})_{\mathbf{E}} = -m\mathbf{I}. \tag{30}$$

Since we are interested only in the linear theory, we assume also that $\theta - \theta_0 \ll \theta_0$ so that the temperature equation (21) becomes

$$\rho_0 c_d\partial_t\theta = -\nabla\cdot\boldsymbol{\mu} + \omega - \theta_0 m\partial_t e. \tag{31}$$

The generalized Fourier's law (3) then gives

$$\rho_0 c_d\partial_t\theta = H\nabla^2\theta - \rho_0 K\partial_t^2 e - m\theta_0\partial_t e + 2\mu K\nabla\cdot(\nabla\cdot\hat{\mathbf{E}}) + \omega + \rho_0 K\nabla\cdot\boldsymbol{f}. \tag{32}$$

The momentum equation (5) becomes

$$\rho_0\partial_t^2\boldsymbol{u} = (\lambda + \mu)\nabla(\nabla\cdot\boldsymbol{u}) + \mu\nabla^2\boldsymbol{u} - m\nabla(\theta - \theta_0) + \rho\boldsymbol{f}. \tag{33}$$

The pair of equations (32) and (33) are the equations for thermo-elastic problems where the coupling is much stronger than one gets with Fourier's law, $K = 0$ in (32). Compare this development with that in Boley and Weiner [17], Fung [18], or Parkus [19]; note particularly that no discussion of entropy variation has been necessary for the development here.

4. Small stress variation

In the special case of small stress variation from unstressed equilibrium one has that $\mathbf{T} \cong 0$ whence, from (27), $\hat{\mathbf{E}} = 0$ and

$$e = 3\alpha(\theta - \theta_0). \tag{34}$$

Now, $\mathbf{T}$ constant means that (14) implicitly defines $\mathbf{E}$ as a function of θ and

$$\partial_t\mathbf{E} = \partial_\theta\mathbf{E})_{\mathbf{T}}\partial_t\theta.$$

But then in (21),

$$\theta\,\mathrm{tr}\,[\partial_\theta\mathbf{T})_{\mathbf{E}}\partial_t\mathbf{E}] = -\rho_0\theta\,\mathrm{tr}\,[\partial_{\mathbf{E}}\eta)_\theta\partial_\theta\mathbf{E})_{\mathbf{T}}]\partial_t\theta = -\rho_0(c_s - c_d)\partial_t\theta$$

by (19). Substitution into (21), gives the temperature equation

$$\rho_0 c_s\partial_t\theta = -\nabla\cdot\boldsymbol{\mu} + \omega. \tag{35}$$

Using the heat flux expression (3) and (34) one replaces (32) with

$$3\alpha\rho_0 K\partial_t^2\theta + \rho_0 c_s \partial_t \theta = H\nabla^2\theta + \omega + \rho_0 K\nabla\cdot\boldsymbol{f}. \tag{36}$$

Notice that in the process of uncoupling the temperature equation the specific heat for constant stress c_s enters the coefficient of $\partial_t\theta$ just as in the gas dynamics case, [8]. The two specific heats are related by

$$c_s - c_d = \theta\alpha^2(3\lambda + 2\mu)\rho^{-1}. \tag{37}$$

The Vernotte generalization (2) leads to a damped wave equation like (36) if one adds τ times the time derivative of (35) to (35) itself and uses (2) to eliminate the terms in $\boldsymbol{\mu}$, a procedure which lacks intuitive appeal. For further discussion see Achenbach [20], [21].

For the following time harmonic analysis we will ignore the body forces $\boldsymbol{f}$ and internal heat supply ω.

5. Time harmonic waves

Time harmonic displacement and temperature variations with frequency $\omega/2\pi$ propagating in the direction of the unit vector $\boldsymbol{n}$ are given by

$$\boldsymbol{u} = A\boldsymbol{a}\exp[i(k\boldsymbol{x}\cdot\boldsymbol{n} - \omega t)], \tag{38}$$

$$\theta - \theta_0 = \tau\exp[i(k\boldsymbol{x}\cdot\boldsymbol{n} - \omega t)]. \tag{39}$$

Substitution of (38) and (39) into (33) and (32) gives respectively

$$(\mu k^2 - \omega^2\rho)A\boldsymbol{a} + k^2(\lambda + \mu)A(\boldsymbol{n}\cdot\boldsymbol{a})\boldsymbol{n} + ikm\tau\boldsymbol{n} = 0, \tag{40}$$

$$(Hk^2 - i\omega\rho c_d)\tau + (\tfrac{4}{3}\mu Kik^3 + m\theta_0\omega k - i\rho K\omega^2 k)A\boldsymbol{n}\cdot\boldsymbol{a} = 0 \tag{41}$$

for the momentum and temperature relations where we have dropped the subscript on ρ since it is assumed to be constant. Eliminating τ between (41) and (40) one gets

$$\left\{(\mu k^2 - \omega^2\rho)\boldsymbol{a} + k^2(\lambda + \mu)(\boldsymbol{n}\cdot\boldsymbol{a})\boldsymbol{n} + \frac{m^2\theta_0\omega k^2 + i(\tfrac{4}{3}m\mu Kk^4 - \rho mK\omega^2 k^2)}{\omega\rho c_d + iHk^2}(\boldsymbol{n}\cdot\boldsymbol{a})\boldsymbol{n}\right\}A = 0.$$

For a nontrivial displacement the coefficient of A must vanish. Thus

$$(\mu k^2 - \omega^2\rho)\boldsymbol{a} + k^2(\lambda + \mu)(\boldsymbol{n}\cdot\boldsymbol{a})\boldsymbol{n} + \frac{(\lambda + 2\mu)\mathscr{E}\omega k^2 + i(\tfrac{4}{3}\mu m\mathscr{K}k^4 - \rho mK\mathscr{H}^2k^2\omega^2)}{\omega + ik^2}(\boldsymbol{n}\cdot\boldsymbol{a})\boldsymbol{n} = 0 \tag{42}$$

where we have introduced the thermo-elastic coupling coefficient $\mathscr{E}$, the mechano-caloric coupling coefficient $\mathscr{K}$ and the heat diffusivity $\mathscr{H}$ defined by

$$\mathscr{E} = \frac{m^2\theta_0}{(\lambda + 2\mu)\rho c_d}, \quad \mathscr{K} = \frac{K}{\rho c_d}, \quad \mathscr{H} = \frac{H}{\rho c_d}. \tag{43}$$

For transverse wave motion, $\boldsymbol{n}$ is orthogonal to $\boldsymbol{a}$, (42) reduces to

$$\mu k^2 - \omega^2 \rho = 0,$$

and the velocity of the transverse wave $c_T = \omega/k$ satisfies

$$c_T^2 = \mu/\rho.$$

The transverse waves are not influenced by the thermo-elastic nor by the mechano-caloric effects.

The longitudinal waves, $\boldsymbol{n}$ parallel to $\boldsymbol{a}$, yield the condition

$$(\lambda + 2\mu)k^2 - \rho\omega^2 + \frac{(\lambda + 2\mu)\mathscr{E}\omega k^2 + im\mathscr{K}k^2(\frac{4}{3}\mu k^2 - \rho\omega^2)}{\omega + ik^2\mathscr{H}} = 0. \tag{44}$$

Clearly the wave number k is complex in this case. The "velocity" $c = \omega/k$ satisfies

$$c_L^2 - c^2 + \frac{cc_L^2\mathscr{E}}{c + ik\mathscr{H}} + i\frac{m\mathscr{K}k(\frac{4}{3}\mu - \rho c^2)}{\rho(c + ik\mathscr{H})} = 0 \tag{45}$$

where $c_L^2 = (\lambda + 2\mu)/\rho$ is the purely mechanical longitudinal wave velocity. Since c depends on k and since it is obviously complex we see that the longitudinal waves exhibit dispersion and attenuation as expected. From (40) one obtains now that

$$\tau/A = (i\rho/m)\left(\frac{c_L^2 - c^2}{c}\right)\omega.$$

The limiting values for c for high and low frequencies are determined by letting ω go to 0 and ∞ with $c = \omega/k$ remaining bounded. Thus as $\omega \to \infty$, $k \to \infty$ and (45) yields the high frequency limit

$$c_\infty^2 = c_L^2\left(1 - \frac{(3\lambda + 2\mu)B}{3(\lambda + 2\mu)(1 + B)}\right) \tag{46}$$

where $B = m\mathscr{K}/\mathscr{H}$. As $\omega \to 0$, $k \to 0$ and (45) gives the low frequency limit

$$c_0^2 = c_L^2(1 + \mathscr{E}),$$

see [20] page 394 for comparison with the results using Fourier's law. Observe that the mechanical response at high frequencies can be determined by the momentum equation alone provided the stress relation is modified to the adiabatic stress relation, see [8],

$$\mathbf{T} = \frac{\lambda - \frac{2}{3}\mu B}{1 + B}\, e\mathbf{I} + 2\mu\mathbf{E}. \tag{47}$$

The adiabatic stress relation is [18]

$$\mathbf{T} = \left(\lambda + \frac{m^2\theta_0}{\rho c_d}\right)e\mathbf{I} + 2\mu\mathbf{E}$$

so that B must satisfy

$$(3\lambda + 2\mu)\left(\frac{1}{3} + \frac{\alpha^2(3\lambda + 2\mu)\theta_0}{\rho c_d}\right)B = -\frac{m^2\theta_0}{\rho c_d}$$

or, using (37),

$$B = -\frac{\gamma - 1}{\gamma - \frac{2}{3}}$$

where $\gamma = c_s/c_d$. Since, by (37), $\gamma > 1$, then $0 > B > -1$. Since also $4\mu < 3(\lambda + 2\mu)$, then

$$1 < \left(1 + \frac{4\mu}{3(\lambda + 2\mu)} B\right)(1 + B)^{-1}.$$

We see then that the mechano-caloric interaction does not influence the propagation velocity at low frequencies, but it increases the propagation velocities at high frequencies. This is exactly like the gas dynamics case discussed in [8].

With slight modification, (45) can be written as

$$(c^2 - c_L^2)(c^2 + i\omega\mathscr{H}) + c^2 c_L^2 \mathscr{E} + i\omega m(\tfrac{4}{3}\mu - \rho c^2)\mathscr{K}/\rho = 0,$$

a quadratic equation in c^2. Thus, considering ω as a given parameter with c to be determined,

$$c^2 = \tfrac{1}{2}c_L^2(1 + \mathscr{E}) - \frac{i}{2}\omega(\mathscr{H} + m\mathscr{K})$$
$$\pm \tfrac{1}{2}(c_L^2 + i\omega\mathscr{H})\left\{1 + 2\frac{c_L^2 - i\omega\mathscr{H}}{(c_L^2 + i\omega\mathscr{H})^2}(c_L^2\mathscr{E} - i\omega m\mathscr{K})\right.$$
$$\left. + \frac{16 i\omega\mu m\mathscr{K}}{3\rho} + (c_L^2\mathscr{E} - i\omega m\mathscr{K})^2\right\}^{\frac{1}{2}}$$

To first order terms in $\mathscr{E}$ and $\mathscr{K}$

$$c_+^2 = c_L^2(1 + \delta\mathscr{E} + \gamma\mathscr{K}) + \ldots,$$

where $\delta = (1 + i\omega\mathscr{H}/c_L^2)^{-1}$ and

$$\gamma = \left(1 + \frac{4\mu}{3(\lambda + 2\mu)}\right)(c_L^2 + i\omega\mathscr{H})^{-1}$$

whereas

$$c_-^2 = -i\omega\mathscr{H} + \eta\mathscr{E} + \zeta\mathscr{K} + \ldots,$$

where $\eta = i\omega\mathscr{H}c_L^2(c_L^2 + i\omega\mathscr{H})^{-1}$, $\zeta = [\omega^2 m\mathscr{H} + i(4\mu\omega m/3\rho)](c_L^2 + i\omega\mathscr{H})^{-1}$. The principal term, $i\omega\mathscr{H}$, represents heat diffusion; both η and ζ have real parts which represents signal transmission.

6. Conclusions

The above analysis suggests that the distinction between c_d and c_s should be carefully observed in the coupled and uncoupled energy field equations, especially at high temperatures. The mechano-caloric effect provides a stronger coupling between the mechanical and thermal variations through the mechano-caloric coefficient $\mathscr{K}$. The velocity of propagation of heat waves is determined by $\mathscr{K}$, see (36), and the limiting propagation velocity for high frequency waves is increased by a factor determined by $\mathscr{K}$. The mechanical response at high frequencies can be determined from the mechanical relations alone if the stress-strain relation is replaced by an "adiabatic" relation. Experimental studies of the mechano-caloric effect and determination of $\mathscr{K}$ would be very interesting, and an analysis of the impact of the mechano-caloric effect in non-linear and plastic systems should be done; it is possible that effects now perceived to be a result of non-linear behavior, Johnson [22], are due to the mechano-caloric coupling.

REFERENCES

[1] C. Cattaneo, Sur une forme de l'equation de la chaleur eliminant le paradoxe d'une propagation instantanée. *C. R. Acad. Sci. Paris* 247 (1958) 431–433.

[2] P. Vernotte, La véritable équation de la chaleur. *C. R. Acad. Sci. Paris* 246 (1958) 3154–3155.

[3] J. Meixner, On the linear theory of heat conduction. *Arch. Rat. Mech. Anal.* 39 (1970) 108–130.

[4] I. Müller, Toward relativistic thermodynamics. *Arch. Rat. Mech. Anal.* 34 (1969) 259–282.

[5] M. E. Gurtin and A. C. Pipkin, A general theory of heat conduction with finite wave speed. *Arch. Rat. Mech. Anal.* 31 (1968) 113–126.

[6] T. Tokuoka, Thermo-acoustical waves in thermo-plastic materials. *J. Engng. Math.* 8 (1974) 9–22.

[7] E. L. Roetman, Heat conduction with finite signal time. *Int. J. Engng. Sci.* 13 (1975) 699–701.

[8] E. L Roetman, Thermomechanical effect for acoustical propagation in gases. *Letters Appl. and Engng. Sci.* (to appear).

[9] S. R. de Groot and P. Mazur, *Non-equilibrium thermodynamics*, North-Holland: Amsterdam, 1962.

[10] S. Chapman and T. G. Cowling, *The mathematical theory of non-uniform gases*, 3rd ed. Cambridge U.P.: London, 1970.

[11] S. K. Loyalka, Kinetic theory of thermal transpiration and mechano-caloric effect. *J. Chem. Phys.* 55 (1971) 4497–4503.

[12] C. A. Truesdell and R. Toupin, The classical field theories, *Handbuch der Physik III/3* (Edited by S. Flügge) Springer: Berlin, 1965.

[13] D. E. Carlson, Linear thermoelasticity, *Handbuch der Physik VI a/2* (Edited by S. Flügge) Springer: Berlin, 1972.

[14] B. D. Coleman and W. Noll, The thermodynamics of elastic materials with heat conduction and viscosity. *Arch. Rat. Mech. Anal.* 13 (1963) 167–178.

[15] B. D. Coleman and V. J. Mizel, Existence of caloric equations of state in thermodynamics. *J. Chem. Phys.* 40 (1964) 1116–1125.

[16] P. Chadwick, Thermoelasticity. The dynamical theory, *Progress in Solid Mechanics I.* (Edited by I. N. Sneddon and R. Hill) North-Holland: Amsterdam, 1960.

[17] B. A. Boley and J. H. Weiner, *Theory of thermal stresses.* Wiley: New York, 1960.

[18] Y. C. Fung, *Foundations of solid mechanics.* Prentice-Hall: Englewood Cliffs, N.J., 1965.

[19] H. Parkus, *Thermoelasticity.* Blaisdell: Waltham, Mass., 1968.

[20] J. D. Achenbach, *Wave propagation in elastic solids.* North-Holland: Amsterdam, 1973.

[21] J. D. Achenbach, The influence of heat conduction on propagating stress jumps. *J. Mech. Phys. Solids* 16 (1968) 273–282.

[22] A. F. Johnson, Pulse propagation in heat conducting elastic materials. *J. Mech. Phys. Solids* 23 (1975) 55–75.